6.1 Basics, Percentage Change

1. (10183308)

It is given that $(m + 2)$ varies directly as $(an + 1)^2$, where a is a positive constant. When $n = 1$, $m = 30$ and when $n = 2$, $m = 96$.

(a) Express m in terms of n.

(b) Someone claims that the minimum value of m is -2. Do you agree? Explain your answer.

2. (10183421)

It is given that y varies directly as x^3.

(a) If x increases by 20%, find the percentage change in y.

(b) When $x = 1.5$, $y = 27$.

(i) Express y in terms of x.

(ii) Find the value of x when $y = 125$.

(iii) Is y directly proportional to x^2? Explain your answer.

(10 marks)

3.

It is given that a varies directly as b and b varies directly as c^2.

(a) Does a vary directly as c^2? Explain your answer.

(b) It is given that $a : b = 3 : 1$. When $c = 4$, $a = 20$. Find the value of a when $c = 12$.

(6 marks)

4.

It is given that W varies directly as the square root of V.

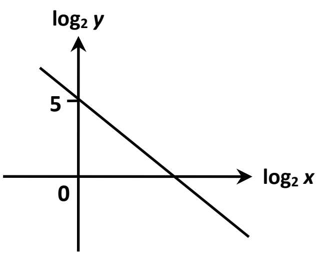
(a) (i) If V decreases by 36%, find the percentage change in W.

(ii) If W increases to 1.6 times its original value, find the percentage change in V.

(b) When V increases by 44%, W increases to 36. Find the original value of W.

(12 marks)

5.



It is given that y varies directly as x^t, where t is a constant, and x and y are positive numbers. The graph in the figure shows the linear relation between $\log_2 x$ and $\log_2 y$. The slope and the intercept on the vertical axis of the graph are $-\frac{1}{4}$ and 5 respectively. Find the variation constant and the value of t.

6.

Suppose $y \propto \frac{1}{x^a}$, where a is a constant and a rational number. When x increases by 25%, y decreases by 36%.

(a) Find the value of a.

(b) When $x = 3$, $y = 8$. Find the value(s) of x when $y = 2$.

(c) If x reduces to $\frac{2}{3}$ of its original value, find the percentage change in y.

(9 marks)

7.*

(a) Find the value of m such that $x - 1$ is a factor of $x^3 + mx^2 + 2x - 6$.

(b) It is given that y varies inversely as $(s^2 + s)$. When $s = 2$, $y = 1$ and when $s = a$, $y = a + 2$, where a is a real number.

  (i) Express y in terms of s.

  (ii) Find the value of a.

  (iii) Find the value(s) of s when $y = \frac{1}{2}$.

(11 marks)

6.2 Joint Variation

8*.

It is given that Q varies directly as P^2 and inversely as $\sqrt{R}$. When $P = 3$ and $R = 36$, $Q = \frac{6}{5}$.

(a) If P decreases by 25% and R decreases by 75%, find the percentage change in Q.

(b) Find the value of Q when $P = 10$ and $R = 144$.

(c) Does Q vary jointly as P and R? Explain your answer.

(8 marks)

9.

Suppose r varies directly as s^2 and inversely as t^2. When $s = 4$ and $t = 2$, $r = 24$.

(a) Express r in terms of s and t.

(b) Find the value of r when $s : t = 5 : 3$.

(c) If $r > 24$ and $t > 4$, must the value of s^2 exceed 65? Explain your answer.

(7 marks)

10.

It is given that m varies jointly as p^a and q, where a is a non-zero constant. When $p = 2$ and $q = 6$, $m = 16$. When $p = 4$ and $q = 3$, $m = 64$.

(a) Find the value of m when $p = 7$ and $q = 9$.

(b) If m increases by 116% and q increases by 25%, find the percentage change in p.

(8 marks)

11*.

Suppose E varies directly as the square of F and inversely as G. Find the percentage change in E in each of the following cases.

(a) F decreases by 25% and G increases by 25%.

(b) F increases to 1.75 times its original value and G decreases by 51%.

(7 marks)

6.3 Partial Variation

12.

It is given that a is the sum of two parts. One part varies directly as b and the other part varies inversely as c. When $b = c = 1$, $a = \frac{10}{3}$. When $b = c = \frac{1}{3}$, $a = 2$.

(a) Express a in terms of b and c.

(b) If b increases by 60% and c decreases by 37.5%, find the percentage change in a.

(c) If $a : b = 4 : 1$, find the value of ac.

(8 marks)

13.

It is given that y is the sum of two parts. One part varies directly as x and the other part varies directly as the square of x. When $x = 2$, $y = 2$ and when $x = 3$, $y = 9$.

(a) Express y in terms of x.

(b) If $y < 44$ and x is a positive integer, how many possible values of x are there? Explain your answer.

(7 marks)

14.

Suppose r partly varies directly as s and partly varies inversely as $\sqrt{t}$. Find the percentage change in r in each of the following cases.

(a) s decreases by 10% and t increases by $\frac{19}{81}$ of its original value.

(b) s increases by 20% and t reduces to $\frac{25}{36}$ of its original value.

(7 marks)

15.

(x) is the sum of two parts. One part varies directly as x^2 and the other part varies directly as x. It is given that $f(4) = 28$ and $f(-2) = 10$.

(a) Find $f(x)$.

(a) Solve the equation $6x^2 - 7x + 1 = 0$.

(b) It is given that $m = \log [f(x)] + (6x^2 - 7x + 1)i$, where x is a real number and $i = \sqrt{-1}$. If m is a purely imaginary number, how many possible values of x are there? Explain your answer.

(8 marks)

16.

$f(x)$ is the sum of two parts. One part varies directly as x^2 and the other part varies directly as x. It is given that $f(-5) = -55$ and $f(3) = 9$.

(a) Find $f(x)$.

(b) Let m be a constant. If the equation $f(x) = m$ has no real roots, find the range of values of m.

(c) A horizontal line L intersects the graph of $y = f(x)$ at only one point P. Q is the foot of perpendicular from P to the x-axis.

(i) Write down the equation of L.

(ii) Find the area of $\triangle PQR$, where R is a point lying on the y-axis.

(8 marks)

17.

When a skydiver with weight m kg opens a parachute after falling s km, the falling speed of the skydiver decreases to v km/h immediately. It is given that v partly varies directly as m and partly varies directly as the square of s. When $m = 75$ and $s = 3$, $v = 10.5$. When $m = 50$ and $s = 4$, $v = 12$.

(a) Express v in terms of m and s.

(b) A skydiver with weight 70 kg starts to fall when he is 4 km above the ground. After he opens a parachute, his falling speed decreases to 9.52 km/h immediately. He claims that his distance above the ground when he opens the parachute is greater than 1.5 km. Do you agree? Explain your answer.

(6 marks)

18.

n workers are needed to build a lighthouse of height h m in t weeks. It is given that t varies directly as h and inversely as n. When $h = 12$ and $n = 20$, $t = 24$.

(a) Express t in terms of h and n.

(b) A construction manager claims that at least 24 workers are needed to build a lighthouse of height 18 m within 30 weeks. Do you agree? Explain your answer.

(c) If the height of the lighthouse increases by 65% and the number of workers increases by 20%, find the percentage increase in the number of weeks needed to build the lighthouse.

(8 marks)

19.

The weight of a wooden cube with total surface area S m^2 is W kg. It is given that W is the sum of two parts. One part varies directly as S and the other part varies directly as $\sqrt{S}$. When $S = 0.25$, $W = 1.625$ and when $S = 1$, $W = 3.5$.

(a) Find the weight of a wooden cube with total surface area 2.56 m^2.

(b) Can a wooden cube with weight 8 kg be put into a cubical box of side 0.8 m? Explain your answer.

(8 marks)

20.

The annual sales amount (Z million dollars) of a company partly varies directly as the advertising expenses (X million dollars) and partly varies inversely as the annual sales amount (Y million dollars) of a competitor. When $X = 0.036$ and $Y = 1$, $Z = 1.25$. When $X = 0.025$ and $Y = 2.14$,
$Z = 0.625$.

(a) Express Z in terms of X and Y.

(b) This year, the advertising expenses of the company are 0.05 million dollars and the annual sales amount of the competitor is 0.5 million dollars.

 (i) Find the annual sales amount of the company this year.

 (ii) The company wants to increase the annual sales amount by 10% next year. Suppose the annual sales amount of the competitor will remain unchanged next year. A manager of the company claims that the increase in the advertising expenses next year should be less than \$50 000. Do you agree? Explain your answer.

(7 marks)

21.

A telecommunication company provides a mobile phone service plan A. It is given that the monthly service fee (\$$C$) is partly constant and partly varies directly as the airtime (n minutes) usage. William now uses service plan A. His airtimes usage in January and February are
840 minutes and 1 280 minutes respectively. His service fees charged in January and February are \$59 and \$70 respectively.

(a) Express C in terms of n.

(b) The telecommunication company provides service plan B. The relation between the monthly service fee and the monthly airtime usage is given as follows:

 I. If the airtime usage in a certain month is not greater than 1 000 minutes, the service fee in that month is \$60.

 II. If the airtime usage in a certain month exceeds 1 000 minutes, the service fee in that month is divided into two parts. One part is the fixed basic monthly fee of \$60. The other part is the service fee charged on the extra airtime usage which exceeds 1 000 minutes. \$5 is charged for every extra hour of airtime usage. Any usage of less than one hour will be counted as one hour.

 William estimates that his average monthly airtime usage in the coming months will be
 1 320 minutes. In order to save money, should he change to plan B? Explain your answer.

(6 marks)

7.1 Basics

1.

Consider a circle $x^2 + y^2 + 6x + 2y - 26 = 0$ with the centre A and a point $B(5, -4)$.

(a) Find the coordinates of A and the radius of the circle.

(b) Does B lie outside the circle? Explain your answer.

(c) If a straight line L is the perpendicular bisector of the line segment joining A and B, find the equation of L.

(9 marks)

2.

The equation of a circle is $\frac{1}{2}x^2 + \frac{1}{2}y^2 + 2x - 3y - \frac{3}{2} = 0$. Two points $A(-1, 1)$ and $B(0, 2)$ are given.

(a) Find the coordinates of the centre and the radius of the circle.

(b) Is the line segment joining A and B inside the circle? Explain your answer.

(c) Someone claims that the perpendicular bisector of the line segment joining A and B passes through the centre of the circle. Do you agree? Explain your answer.

(10 marks)

3.

It is given that C: $x^2 + y^2 + ax + 5y + \frac{25}{2} = 0$ is a real circle, where a is a constant.

(a) Find the range of values of a.

(b) When the smallest positive integral value of a in (a) is taken,

(i) find the coordinates of the centre of C,

(ii) find the equation of the straight line which is parallel to the straight line $5x - 3y + 11 = 0$ and passes through the centre of C.

(6 marks)

4.

It is given that C: $x^2 + y^2 - 14x + 6y + k = 0$ is a real circle, where k is a real number.

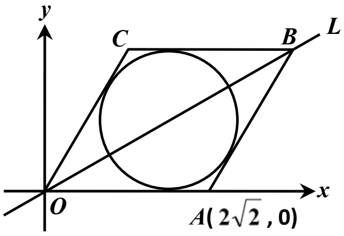
(a) Find the range of values of k.

(b) Take $k = 8$. The circle C and the straight line L: $x + y - 10 = 0$ intersect at two points A and B. M is a point on C such that the area of $\triangle ABM$ is the greatest. Find the coordinates of M.

(9 marks)

7.2 Miscellaneous Questions

5.



In the figure, $OABC$ is a rhombus. The coordinates of A are $(2\sqrt{2}, 0)$. B and C lie in quadrant I. A straight line L passes through O and B. The equation of L is $x - (1+\sqrt{2})y = 0$.

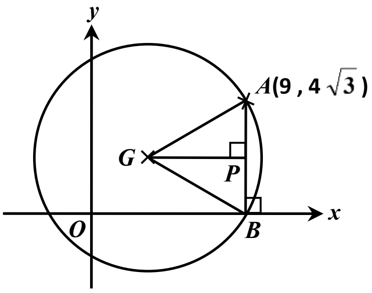
(a) Find the coordinates of B and C.

(b) Find the equation of the inscribed circle of rhombus $OABC$ in the general form.

(*Leave the radical sign '$\sqrt{\ }$' in the answers if necessary.*)

(6 marks)

6.



In the figure, G is the centre of the circle. $A(9, 4\sqrt{3})$ and B are two points on the circle and $AB \perp$ the x-axis, where B lies on the positive x-axis. P is a point on the chord AB such that $GP \perp AB$. It is given that $GA = GB = AB$.

(a) Find the coordinates of G.

(b) Find the equation of the circle in the standard form.

(*Leave the radical sign '$\sqrt{\ }$' in the answers if necessary.*)

(6 marks)

7*.

The circle C_1: $x^2 + y^2 + px + qy + 4p = 0$ passes through $A(4, 0)$ and $B(8, 4)$, where p and q are constants. L_1 is the perpendicular bisector of the line segment joining A and B.

(a) Find the values of p and q.

(b) Find the equation of L_1.

(c) It is given that $M\left(-\frac{1}{2}, k\right)$ is the circumcentre of ΔABD, where the coordinates of D are $(1, 18)$.

 (i) Does D lie inside C_1? Explain your answer.

 (ii) Using the result of (b), find the value of k.

 (iii) Let C_2 be the circumcircle of ΔABD. Find the equation of C_2 in the standard form.

(12 marks)

8.

A circle S passes through three points $A(1 , 2)$, $B(5 , 6)$ and $C(5 , -8)$.

(a) Find the equation of S in the general form.

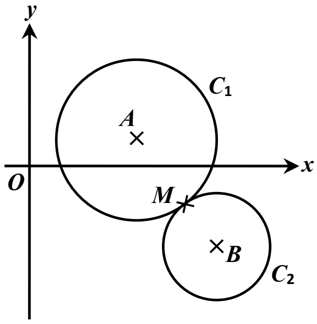
(b) The straight line $y = mx$ and S intersect at two points $P(x_1 , y_1)$ and $Q(x_2 , y_2)$, where $x_1 > 0$ and $x_2 > 0$.

  (i) Show that $x_1x_2 = \dfrac{7}{1+m^2}$.

  (ii) Hence, find the value of $OP \times OQ$, where O is the origin.

(10 marks)

9*.



In the figure, circles C_1 and C_2 touch each other externally at M. The centres of C_1 and C_2 are A and $B(7 , -3)$ respectively. The equation of C_1 is $x^2 + y^2 - 8x - 2y + 8 = 0$.

(a) Find the coordinates of the centre and the radius of C_1.

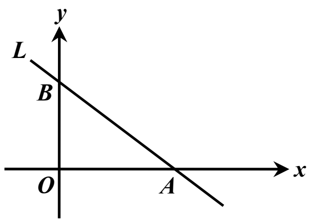
(b) Find the equation of C_2 in the standard form.

(c) Find the coordinates of M.

(d) If a straight line L is the common tangent to C_1 and C_2 at M, find the equation of L.

(10 marks)

10.



In the figure, the equation of straight line L is $3x + 4y - 36 = 0$. L intersects the x-axis and the y-axis at A and B respectively.

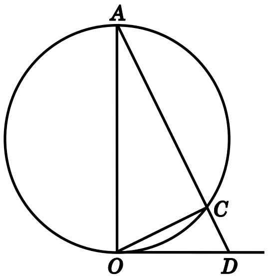
(a) Find the coordinates of A and B.

(b) Find the equation of the circumcircle of $\triangle OAB$ in the standard form.

(c) Find the equation of the inscribed circle of $\triangle OAB$ in the general form.

(10 marks)

11.



In the figure, A, O and C are three points on the circle. AC is produced to a point D such that OD touches the circle at O. AO is a diameter of the circle.

(a) Prove that $\triangle ADO \sim \triangle AOC$.

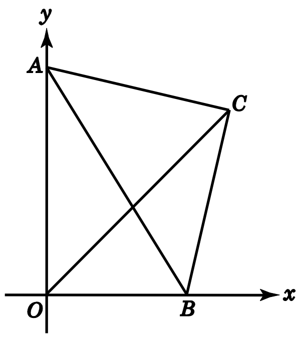
(b) A rectangular coordinate system, with O as the origin, is introduced in the figure so that the coordinates of A and D are $(0, 10)$ and $(5, 0)$ respectively.

  (i) Find the coordinates of C.

  (ii) Find the equation of a circle with AC as a diameter in the general form.

(11 marks)

12.



In the figure, $OACB$ is a quadrilateral. The coordinates of A and B are $(0, 16)$ and $(10, 0)$ respectively. The in-centre of $\triangle OAB$ lies on OC and $\angle BAC = 45°$.

(a) Prove that O, A, C and B are concyclic.

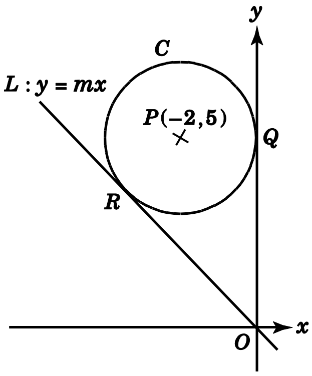
(b) Find the equation of the circumcircle of quadrilateral $OACB$ in the standard form.

(c) A straight line L touches the circle obtained in (b) at C. Is L parallel to AB? Explain your answer.

(10 marks)

13.

In the figure, the circle C with centre $P(-2, 5)$ touches the y-axis at Q. The straight line L: $y = mx$ passes through the origin O. L touches the circle C at point R.



(a) Find the equation of the circle C in the general form.

(b) Find the value of m.

(c) (i) Prove that O, Q, P and R are concyclic.

  (ii) Find the equation of the circle that passes through points O, Q, P and R in the general form.

(13 marks)

7.3 Intersections of circles and straight lines

14.

The equation of straight line L is $y = 6x + k$, where k is a constant. The equation of circle C is $x^2 + y^2 - 6x + 2y - 7 = 0$.

(a) If L cuts C into two equal parts, find the value of k.

(b) If L intersects C at two points, find the range of values of k.

(*Leave the radical sign '* $\sqrt{\ }$ *' in the answers.*)

(7 marks)

15.

The equation of a circle is $x^2 + y^2 - 8x + 4y + 15 = 0$. Which of the following are true?

I. The circle is a real circle.

II. The circle passes through point $(6\ ,\ -1)$.

III. The straight line $y = 11 - 2x$ does not intersect the circle.

A. I and II only B. I and III only C. II and III only D. I, II and III

16.

The coordinates of the centre of a circle are $(0\ ,\ 2)$ and the radius is $\sqrt{3}$. If the straight line L: $y = \sqrt{3}x + k + 2$ intersects the circle at two points, find the range of values of k.

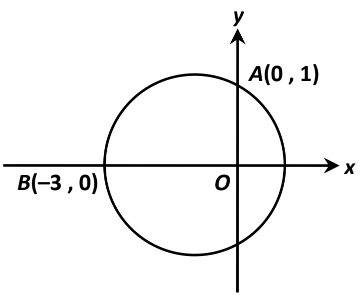
A. $\sqrt{3} < k < 3\sqrt{3}$

B. $-2\sqrt{3} < k < 2\sqrt{3}$

C. $k < -\sqrt{3}$ or $k > \sqrt{3}$

D. $k < -2\sqrt{3}$ or $k > 2\sqrt{3}$

17.



In the figure, the centre G of the circle lies on the x-axis. The circle intersects the negative x-axis at $B(-3\ ,\ 0)$ and intersects the positive y-axis at $A(0\ ,\ 1)$. Find the equation of the circle.

A. $\left(x+\dfrac{4}{3}\right)^2 + y^2 = \dfrac{5}{3}$ B. $\left(x+\dfrac{4}{3}\right)^2 + y^2 = \dfrac{25}{9}$

C. $x^2 + \left(y+\dfrac{4}{3}\right)^2 = \dfrac{5}{3}$ D. $x^2 + \left(y+\dfrac{4}{3}\right)^2 = \dfrac{25}{9}$

18*.

If the straight line $x + 2y = k$ intersects the circle $x^2 + y^2 - 9x - 4y + 5 = 0$ at two points A and B, find the x-coordinate of the mid-point of AB.

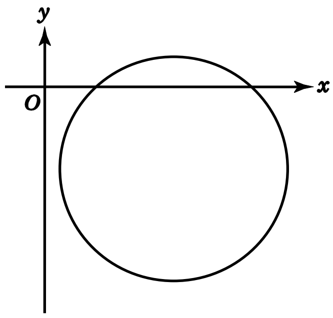
A. $\dfrac{k-7}{5}$ B. $\dfrac{k+14}{5}$ C. $\dfrac{2k-14}{5}$ D. $\dfrac{2k+28}{5}$

19.

It is given that the straight line $\dfrac{x}{a} + \dfrac{y}{b} = 1$ touches the circle $(x - a)^2 + (y - b)^2 = k$, where a and b are non-zero constants and $k > 0$. Express k in terms of a and b.

A. $a^2 + b^2$ B. $\dfrac{a^2b^2}{a^2+b^2}$ C. a^2b^2 D. $(a + b)^2$

20.



The equation of the circle in the figure is $(x - h)^2 + (y + k)^2 = r^2$, where h, k and r are positive constants. Which of the following are true?

I. $h - k > 0$ II. $r - h < 0$ III. $r - k > 0$

A. I and II only B. I and III only C. II and III only D. I, II and III

21.

If the straight line $ax + y = 0$ is a tangent to the circle $(x - 4)^2 + y^2 = 12$, find the value(s) of a.

A. $\sqrt{3}$ B. $-\sqrt{3}$ C. $\sqrt{3}$ or $-\sqrt{3}$ D. 3 or –3

22.

The circle $x^2 + y^2 - 10x + 5y + 20 = 0$ and the straight line $y = mx$ intersect at two distinct points. Find the range of values of m.

A. $-2 < m < \dfrac{2}{11}$

B. $-\dfrac{2}{11} < m < 2$

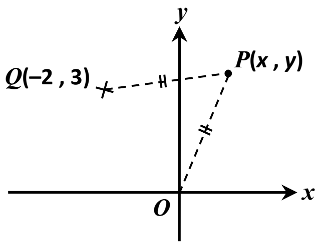
C. $m < -2$ or $m > \dfrac{2}{11}$

D. $m < -\dfrac{2}{11}$ or $m > 2$

Think carefully whether each of the following questions belongs to type 1 - 8.
If not, remember how to write the first line.

1.
In a rectangular coordinate plane, a moving point $P(x , y)$ maintains a fixed distance of 5 from a point $Q(-3 , 2)$.
(a) Find the equation of the locus of P.
(b) Is $(-7 , -1)$ a point on the locus of P? Explain your answer.
(c) It is given that the equation of circle C is $x^2 + y^2 + 6x - 4y - 18 = 0$. Describe the geometric relationship between the locus of P and C.

(7 marks)

2.


In the figure, $Q(-2 , 3)$ is a point in a rectangular coordinate plane. A moving point $P(x , y)$ maintains an equal distance from Q and the origin O, i.e. $PQ = OP$.
(a) Find the equation of the locus of P.
(b) Is the straight line L: $3x + 2y + 17 = 0$ perpendicular to the locus of P? Explain your answer.

(5 marks)

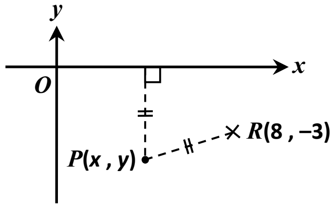
3*.
$A(5 , -3)$ and $B(3 , -1)$ are two points in a rectangular coordinate plane. A point $P(x , y)$ moves such that $PA = 2PB$.
(a) Find the equation of the locus of P.
(b) Describe the locus of P.

(6 marks)

4.
$A(9 , 2)$ and $B(-1 , -6)$ are two points in a rectangular coordinate plane. A point $P(x , y)$ moves such that $AP : PB = 1 : \sqrt{3}$.
(a) Find the equation of the locus of P.
(b) Find the area enclosed by the locus of P in terms of π.

(5 marks)

5.



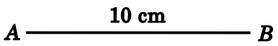
In the figure, $R(8 , -3)$ is a point in a rectangular coordinate plane. A moving point $P(x , y)$ maintains an equal distance from R and the x-axis.

(a) Describe the locus of P.

(b) Find the equation of the locus of P.

(5 marks)

6.



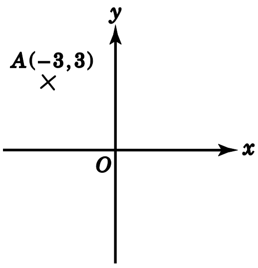
In the figure, the length of the line segment AB is 10 cm.

(a) A moving point P maintains a fixed distance of 5 cm from the mid-point of AB. Sketch and describe the locus of P.

(b) A moving point Q maintains a fixed distance of 5 cm from the line segment AB. Sketch and describe the locus of Q.

(5 marks)

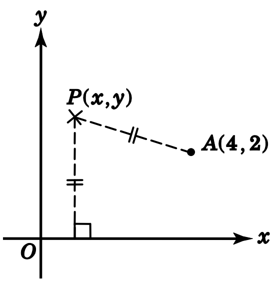
7.



In the figure, $A(-3 , 3)$ is a point in a rectangular coordinate plane. A moving point $P(x , y)$ maintains a fixed distance from A. If the locus of P passes through the origin O, find the equation of the locus of P.

(4 marks)

8.



In the figure, $A(4 , 2)$ is a point in a rectangular coordinate plane. A moving point $P(x , y)$ maintains an equal distance from A and the x-axis. Find the equation of the locus of P.

(4 marks)

9.

$M(-1 , -1)$ and $N(3 , 2)$ are two points in a rectangular coordinate plane. A moving point P maintains $\angle MNP = 90°$ with M and N. A point Q moves such that $MQ = NQ$.

(a) Find the equation of the locus of P.

(b) Find the equation of the locus of Q.

(c) Describe the geometric relationship between the loci of P and Q.

(8 marks)

10.

$A(-11 , 5)$ and $B(3 , 7)$ are two points in a rectangular coordinate plane. A point $P(x , y)$ moves such that $AP^2 + BP^2 = AB^2$. Denote the locus of P by Γ.

(a) Find the equation of Γ.

(b) Straight line L is the perpendicular bisector of the line segment AB.

  (i) Find the equation of L.

  (ii) Let F be a point on Γ. L and Γ intersect at two points R and S. Find the greatest possible area of $\triangle FRS$.

(8 marks)

11.

$A(1 , 2)$ and $B(4 , 4)$ are two points in a rectangular coordinate plane. A moving point $P(x , y)$ maintains an equal distance from A and B. A moving point Q lies below AB such that the area of $\triangle QAB$ is equal to the area of $\triangle OAB$, where O is the origin.

(a) Find the equation of the locus of P.

(b) Find the equation of the locus of Q.

(c) Describe the geometric relationship between the loci of P and Q.

(8 marks)

12.

$G(3 , -4)$ is a point in a rectangular coordinate plane. A moving point $P(x , y)$ maintains an equal distance from a point G and the straight line L: $y = 12$.

(a) Find the equation of the locus of P.

(b) Find the coordinates of the vertex of the locus of P.

(c) R is a point on the locus of P. R' is the reflection image of R with respect to L. Suppose the distance between R and R' is the shortest.

  (i) Describe the geometric relationship between the three points G, R and R'.

  (ii) If F is a point in the rectangular coordinate plane such that the area of $\triangle FGR$ is 60, find the area of $\triangle FRR'$.

(10 marks)

13.

In a rectangular coordinate plane, M is the centre of the circle C: $x^2 + y^2 - 6x - 20y + 84 = 0$. P is a moving point on C. A moving point $Q(x , y)$ maintains $QN = 3$ with the point $N(9 , 2)$. Denote the locus of Q by Γ.

(a) Find the equation of Γ.

(b) Show that C and Γ do not intersect.

(c) Find the smallest possible length of PQ.

(d) When P is nearest to Q,

  (i) describe the geometric relationship between the four points M, P, Q and N,

  (ii) find the ratio of the area of $\triangle OMQ$ to the area of $\triangle OPN$, where O is the origin.

(10 marks)

14.

$M(1 , 0)$ and $N(5 , 6)$ are two points in a rectangular coordinate plane. If $P(x , y)$ moves such that it satisfies each of the following conditions, find the equation of the locus of P.

(a) $PM = PN$

(b) $PM : PN = 3 : 1$

(6 marks)

15.

$A(0 , 3)$ and $B(4 , 5)$ are two points in a rectangular coordinate plane. The locus of a moving point $P(x , y)$ is formed by the centres of all the circles passing through both points A and B.

(a) Sketch and describe the locus of P.

(b) Find the equation of the locus of P.

(c) Suppose the locus of P intersects the x-axis at point C. A moving point $M(x , y)$ maintains a fixed distance from C and the locus of M passes through A. Find the equation of the locus of M.

(10 marks)

16.

The coordinates of point A and point B are $(2 , -6)$ and $(5 , 3)$ respectively. A is rotated anticlockwise about the origin O through 90° to A'. B' is the reflection image of B with respect to the x-axis. $P(x , y)$ is a moving point in the rectangular coordinate plane such that $\angle A'PB' = 90°$.

(a) Write down the coordinates of A' and B'.

(b) Find the equation of the locus of P.

(c) Does the locus of P intersect the x-axis? Explain your answer.

(10 marks)

17.

In a rectangular coordinate plane, the y-intercepts of straight lines L_1 and L_2 are 3 and -2 respectively. The x-intercept of L_1 is 2 and $L_1 // L_2$. A moving point P maintains an equal distance from L_1 and L_2. Denote the locus of P by Γ.

(a) (i) Describe Γ.

(ii) Find the equation of Γ.

(b) The equation of circle C is $x^2 + y^2 + 2x - 4y + 1 = 0$ and G is the centre of C.

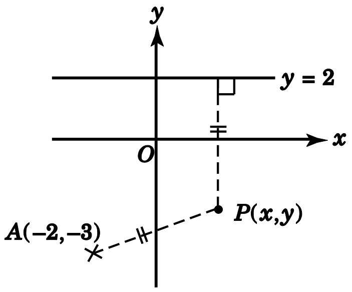
(i) Does Γ pass through G? Explain your answer.

(ii) L_1 intersects C at A and B, where the x-coordinate of B is smaller than that of A. Γ intersects C at D and E, where the x-coordinate of E is smaller than that of D. Find the ratio of the area of $\triangle ADG$ to the area of $\triangle BDE$.

(9 marks)

18.

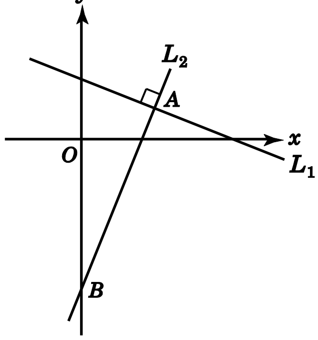
In the figure, $A(-2 , -3)$ is a point in a rectangular coordinate plane. A point $P(x , y)$ moves such that it maintains an equal distance from A and the line $y = 2$. Find the equation of the locus of P.



(4 marks)

19.

In the figure, straight lines L_1: $2x + 5y - 20 = 0$ and L_2 are perpendicular to each other. L_1 and L_2 intersect at point A. L_2 intersects the y-axis at B and passes through $(7 , 7)$.



(a) Find the equation of L_2.

(b) A moving point $P(x , y)$ maintains an equal distance from A and B.

(i) Describe the locus of P.

(ii) Find the equation of the locus of P.

(12 marks)